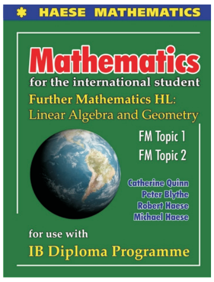


ERRATA

MATHEMATICS FOR THE INTERNATIONAL STUDENT

FURTHER MATHEMATICS HL: Linear Algebra and Geometry

First edition - 2015 first reprint

The following erratum was made on 12/Sep/2017

page 272 **Worked Solutions EXERCISE 1K.1 Question 4 a,** should not exclude circles being considered as ellipses:

4 From **3**, $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{|\mathbf{A}|}\begin{pmatrix} dx' - by' \\ -cx' + ay' \end{pmatrix}$

$\therefore \quad x^2 + y^2 = 1$ becomes

$$\left(\frac{dx' - by'}{|\mathbf{A}|}\right)^2 + \left(\frac{-cx' + ay'}{|\mathbf{A}|}\right)^2 = 1$$

$\therefore \quad d^2x'^{\,2} - 2bdx'y' + b^2y'^{\,2} + c^2x'^{\,2} - 2acx'y' + a^2y'^{\,2}$
$= |\mathbf{A}|^2$

$\therefore \quad (c^2+d^2)x'^{\,2} - 2(ac+bd)x'y' + (a^2+b^2)y^2 = |\mathbf{A}|^2$

a An ellipse has the form $Ax^2 + By^2 = C$
So, we require $-2(ac+bd) = 0$

b A circle has the form $Ax^2 + By^2 = C$ where $A = B$.
So, we require $-2(ac+bd) = 0$
and $(c^2+d^2) = (a^2+b^2)$

The following erratum was made on 31/Jul/2017

page 267 **Worked Solutions EXERCISE 1J.2 Question 4 c,** should read:

4 c $\mathrm{T}\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} y \\ -z \\ x+y \end{pmatrix}$

$$= x\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + z\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

So, $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

or $\mathrm{T}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\mathrm{T}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$,

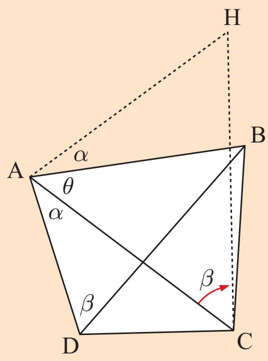
and $\mathrm{T}\left(\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

So, $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$

The following erratum was made on 28/Jul/2017

page 179 **SECTION 2J Proof for the converse for Ptolemy's theorem for cyclic quadrilaterals,** replace entirely with:

Proof:



Let $C\widehat{A}D = \alpha$, $A\widehat{D}B = \beta$, and $B\widehat{A}C = \theta$.

Choose a point H on the same side of AC as B, such that $B\widehat{A}H = \alpha$ and $A\widehat{C}H = \beta$.

Now $C\widehat{A}H = D\widehat{A}B = \alpha + \theta$

and $A\widehat{C}H = A\widehat{D}B = \beta$

$\therefore$ $\triangle$s ACH and ADB are equiangular and therefore similar.

$$\therefore \quad \underbrace{\frac{CH}{DB} = \frac{AC}{AD}}_{(1)} \underbrace{\;= \frac{AH}{AB}}_{(2)}$$

Rearranging (1), $CH = \dfrac{AC.DB}{AD}$ .... (3)

Given (2) and that $B\widehat{A}H = D\widehat{A}C = \alpha$, in $\triangle$s BAH and DAC we have two sides in the same ratio and the included angle between them equal.

$\therefore$ $\triangle$s BAH and DAC are similar.

$$\therefore \quad \frac{BH}{CD} = \frac{AB}{AD}$$

$$\therefore \quad BH = \frac{AB.CD}{AD} \quad .... (4)$$

Using (3) and (4),

$$\begin{aligned} CH - BH &= \frac{AC.DB}{AD} - \frac{AB.CD}{AD} \\ &= \frac{AC.BD - AB.CD}{AD} \\ &= \frac{BC.AD}{AD} \quad \{\text{by the assumption of the converse}\} \\ &= BC \end{aligned}$$

$\therefore$ $BC + BH = CH$, which can only be true if B, C, and H are collinear.

$\therefore$ $A\widehat{C}B = \beta$

$\therefore$ $A\widehat{C}B = A\widehat{D}B$ are equal angles subtended by [AB].

$\therefore$ ABCD is a cyclic quadrilateral.

Note: this pushes questions 3 and 4 in Exercise 2J, from page 180 to page 181.

The following errata were made on or before 09/Jan/2017

page 65 **SECTION 1I Subspaces,** third fact-box should read:

Every subspace of $\mathbb{R}^n$ contains the zero vector $\mathbf{0}$.

page 184 **SECTION 2K Example 17,** question should read:

Example 17

LMN is a triangle. X is on [LM], Y is on [MN], and Z is on [NL]. LX = 4 cm, MY = 3 cm, NZ = 5 cm, YN = 6 cm, ZL = 2 cm, and XM = 5 cm.

Prove that [MZ], [NX], and [LY] are concurrent.

The following errata were made on or before 06/Dec/2016

page 42 **EXERCISE 1F.1 Question 2 d,** should read:

2 Find the determinant of each of the following matrices. Hence find the inverse, if it exists.

a $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ **c** $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ **d** $\begin{pmatrix} 5 & 0 \\ 3 & 0 \end{pmatrix}$ **e** $\begin{pmatrix} a & a \\ -a & 1 \end{pmatrix}$

page 47 **EXERCISE 1F.3 Question 4,** should read:

4 For what values of a and b does $\begin{pmatrix} a^2 & 1 & 1 \\ 0 & a & b \\ 1 & 0 & 0 \end{pmatrix}$ have an inverse?

page 48 **EXERCISE 1F.4 Question 3,** should read:

3 If $\mathbf{A}$ is 3×3, explain why $|k\mathbf{A}| = k^3 |\mathbf{A}|$.

page 68 **EXAMPLE 30 Solution,** second to last line should read:

Since $|\mathbf{A}| = 0$, $\mathbf{A}^{-1}$ does not exist.

$\therefore$ $\mathbf{Ac} = \mathbf{x}$ does not have a unique solution $\mathbf{c} = \mathbf{A}^{-1}\mathbf{x}$ for each vector $\mathbf{x}$.

Thus $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ do not span $\mathbb{R}^3$.

page 100 **EXAMPLE 54 Answer part b,** last 3 lines should read:

$\Rightarrow \quad -x' + 3y' = 2x' + 2y' - 4$

$\Rightarrow \quad 3x' - y' = 4$

Hence $y = 2x - 1 \xrightarrow{T} 3x - y = 4$

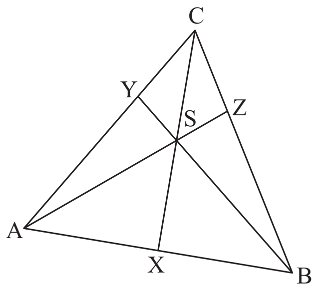
page 112 **EXERCISE 1K.4 Question 8 c,** should read:

8 **a** Find the transformation matrix for a vertical stretch with scale factor k.

b Find the single transformation matrix for a vertical stretch with scale factor 2, followed by a reflection in the line $y = -2x$.

c What effect does the composition of transformations in **b** have on sense and area?

page 185 **EXERCISE 2K.1 Question 3,** should be replaced entirely with:

3 

In the diagram, BZ : ZC = 2 : 1 and AY : YC = 3 : 2.

a Find the ratio in which X divides [AB].

b Find the ratio in which S divides [AZ].

page 242 **Worked Solutions EXERCISE 1F.1 Question 7 b,** should read:

7 b $\mathbf{A} = \begin{pmatrix} k-5 & -3 \\ 2 & k \end{pmatrix}$

$\therefore \det \mathbf{A} = (k-5)(k) - (-3)(2)$
$= k^2 - 5k + 6$
$= (k-3)(k-2)$

$$\mathbf{A}^{-1} = \frac{1}{(k-3)(k-2)}\begin{pmatrix} k & 3 \\ -2 & k-5 \end{pmatrix}$$
$$= \begin{pmatrix} \dfrac{k}{(k-3)(k-2)} & \dfrac{3}{(k-3)(k-2)} \\ \dfrac{-2}{(k-3)(k-2)} & \dfrac{k-5}{(k-3)(k-2)} \end{pmatrix}$$

$\therefore$ $\mathbf{A}^{-1}$ exists provided $(k-3)(k-2) \neq 0$

$\therefore$ $k \neq 3$ or 2

page 273 **Worked Solutions EXERCISE 1K.2 Question 2 c,** should read:

2 c $\mathbf{A} = \begin{pmatrix} -\frac{5}{13} & \frac{12}{13} \\ -\frac{12}{13} & -\frac{5}{13} \end{pmatrix}$ where $|\mathbf{A}| = 1$

Since $\mathbf{A}$ has the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$, $\mathbf{A}$ is a rotation matrix.

If the angle of rotation is θ, $\cos\theta = -\frac{5}{13}$ and $\sin\theta = -\frac{12}{13}$.

$\therefore$ $\tan\theta = \frac{12}{5}$ and $-\pi < \theta < -\frac{\pi}{2}$

$\therefore$ $\theta = \arctan\left(\frac{12}{5}\right)$

$\therefore$ the transformation is a clockwise rotation about O through $\pi - \arctan\left(\frac{12}{5}\right)$ (since $\cos\theta, \sin\theta < 0$).

page 274 **Worked Solutions EXERCISE 1K.2 Question 7 a,** should read:

7 $m = \tan\alpha = -2$

Now $\cos 2\alpha = \dfrac{1-m^2}{1+m^2}$ and $\sin 2\alpha = \dfrac{2m}{1+m^2}$

$\therefore$ $\cos 2\alpha = -\frac{3}{5}$ and $\sin 2\alpha = -\frac{4}{5}$

S has matrix $\mathbf{A} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$

a S has equations $x' = \dfrac{-3x-4y}{5}$
$y' = \dfrac{-4x+3y}{5}$

Thus $(-4, 2) \xrightarrow{S} \left(\dfrac{-3(-4)-4(2)}{5}, \dfrac{-4(-4)+3(2)}{5}\right)$

So, $(-4, 2) \xrightarrow{S} \left(\frac{4}{5}, \frac{22}{5}\right)$

pages 277 and 278 **Worked Solutions EXERCISE 1K.4 Question 2 b,** should be broken into two parts and read:

2 **a** T_1 is a reflection in $y = x$.

T_2 is a rotation about O through $\frac{\pi}{3}$.

T_1 has matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

T_2 has matrix $\mathbf{B} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$.

$\{\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\}$

b **i** $T_1 \circ T_2$ has matrix **AB** where

$$\mathbf{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

and $|\mathbf{AB}| = |\mathbf{A}|\,|\mathbf{B}| = -1 \times 1 = -1$

and **AB** has form $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$.

$\therefore$ $T_1 \circ T_2$ is a reflection in $y = (\tan\alpha)x$

where $\cos 2\alpha = \frac{\sqrt{3}}{2}, \quad \sin 2\alpha = \frac{1}{2}$

$\therefore \ 2\alpha = \frac{\pi}{6}$

$\therefore \ \alpha = \frac{\pi}{12}$

$\therefore$ $T_1 \circ T_2$ is a reflection in $y = (\tan\frac{\pi}{12})x$.

ii $T_2 \circ T_1$ has matrix **BA** where

$$\mathbf{BA} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

and $|\mathbf{BA}| = |\mathbf{B}|\,|\mathbf{A}| = 1 \times -1 = -1$

and **BA** has form $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$.

$\therefore$ $T_2 \circ T_1$ is a reflection in $y = (\tan\alpha)x$

where $\cos 2\alpha = -\frac{\sqrt{3}}{2}, \quad \sin 2\alpha = \frac{1}{2}$

$\therefore \ 2\alpha = \frac{5\pi}{6}$

$\therefore \ \alpha = \frac{5\pi}{12}$

$\therefore$ $T_2 \circ T_1$ is a reflection in $y = (\tan\frac{5\pi}{12})x$.

c As $T_1 \circ T_2$ and $T_2 \circ T_1$ represent different transformations,

$T_1 \circ T_2 \neq T_2 \circ T_1$.

page 278 **Worked Solutions EXERCISE 1K.4 Questions 3 and 4,** should read:

3 **a** T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, and

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$.

$\therefore$ $T_B \circ T_A$ has matrix **BA** where

$$\mathbf{BA} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos\phi\cos\theta - \sin\phi\sin\theta & -\sin\theta\cos\phi - \cos\theta\sin\phi \\ \sin\phi\cos\theta + \cos\phi\sin\theta & -\sin\phi\sin\theta + \cos\phi\cos\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) \\ \sin(\theta+\phi) & \cos(\theta+\phi) \end{pmatrix}$$

which is the matrix for a rotation about O through $(\theta + \phi)$.

b T_A is a reflection in $y = (\tan\alpha)x$ and
T_B is a reflection in $y = (\tan\beta)x$.

T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{pmatrix}$.

Now $T_B \circ T_A$ has matrix **BA** where

BA
$$= \begin{pmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{pmatrix}\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos 2\beta\cos 2\alpha + \sin 2\beta\sin 2\alpha & \cos 2\beta\sin 2\alpha - \sin 2\beta\cos 2\alpha \\ \sin 2\beta\cos 2\alpha - \cos 2\beta\sin 2\alpha & \sin 2\beta\sin 2\alpha + \cos 2\beta\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos(2\beta - 2\alpha) & -\sin(2\beta - 2\alpha) \\ \sin(2\beta - 2\alpha) & \cos(2\beta - 2\alpha) \end{pmatrix}$$

which is the matrix for a rotation about O through an angle of $2(\beta - \alpha)$.

$\therefore$ $T_B \circ T_A$ is a rotation about O through $2(\beta - \alpha)$.

4 Let T_A be a reflection in $y = (\tan\alpha)x$, and
let T_B be a rotation about O through θ.

T_A has matrix $\mathbf{A} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

Now $T_B \circ T_A$ has matrix **BA** where

BA
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta\cos 2\alpha - \sin\theta\sin 2\alpha & \cos\theta\sin 2\alpha + \sin\theta\cos 2\alpha \\ \sin\theta\cos 2\alpha + \cos\theta\sin 2\alpha & \sin\theta\sin 2\alpha - \cos\theta\cos 2\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta + 2\alpha) & \sin(\theta + 2\alpha) \\ \sin(\theta + 2\alpha) & -\cos(\theta + 2\alpha) \end{pmatrix}$$

which is the matrix for a reflection in the line
$y = \left[\tan\left(\frac{\theta}{2} + \alpha\right)x\right]$

$\therefore$ $T_B \circ T_A$ is a reflection in $y = \left[\tan\left(\frac{\theta}{2} + \alpha\right)x\right]$.

page 278 **Worked Solutions EXERCISE 1K.4 Question 6,** first 5 lines should read:

6 T_A is unknown with matrix **A**.
T_B is a clockwise rotation about O through $\frac{\pi}{2}$.
$T_B \circ T_A$ is a reflection in $y = \frac{1}{2}x$.

T_B has matrix $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

For $T_B \circ T_A$, $\tan\alpha = \frac{1}{2}$

page 279 **Worked Solutions EXERCISE 1K.4 Questions 7 and 8,** should read:

7 $\mathrm{T_A}$ is a reflection in $y = -x$.

$\mathrm{T_B}$ is a rotation of $\frac{3\pi}{4}$ about O.

$\mathrm{T_A}$ has matrix $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

$\mathrm{T_B}$ has matrix $\mathbf{B} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$.

$\{\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}, \quad \sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}\}$

$\mathrm{T_B} \circ \mathrm{T_A}$ has matrix $\mathbf{BA}$ where

$$\mathbf{BA} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathrm{T_B} \circ \mathrm{T_A}$ has equations $x' = \dfrac{x+y}{\sqrt{2}}$

$y' = \dfrac{x-y}{\sqrt{2}}$

and $(\mathbf{BA})^{-1} = \mathbf{BA}$ {as $\mathbf{BA}$ is a reflection}

$\therefore\ x = \dfrac{x'+y'}{\sqrt{2}}, \quad y = \dfrac{x'-y'}{\sqrt{2}}$

$$\therefore\ y = 2x - 1 \xrightarrow{\mathrm{T_B} \circ \mathrm{T_A}} \frac{x'-y'}{\sqrt{2}} = 2\left(\frac{x'+y'}{\sqrt{2}}\right) - 1$$

$$\Rightarrow\ x' - y' = 2x' + 2y' - \sqrt{2}$$

$$\Rightarrow\ x' + 3y' = \sqrt{2}$$

$$\therefore\ y = 2x - 1 \xrightarrow{\mathrm{T_B} \circ \mathrm{T_A}} x + 3y = \sqrt{2}$$

8 **a** $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$

b $\mathrm{T_A}$ is a vertical stretch of factor 2.

$\mathrm{T_B}$ is a reflection in $y = -2x$.

$\mathrm{T_A}$ has matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

$\mathrm{T_B}$ has matrix $\mathbf{B} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$.

$\{\tan\alpha = -2 \quad \therefore\ \cos 2\alpha = \dfrac{1-4}{1+4} = -\frac{3}{5},$

$\sin 2\alpha = \dfrac{-4}{1+4} = -\frac{4}{5}\}$

$$\mathrm{T_B} \circ \mathrm{T_A} \text{ has matrix } \mathbf{BA} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{3}{5} & -\frac{8}{5} \\ -\frac{4}{5} & \frac{6}{5} \end{pmatrix}$$

c $\mathrm{T_B} \circ \mathrm{T_A}$ has $|\mathbf{BA}| = |\mathbf{B}|\,|\mathbf{A}|$

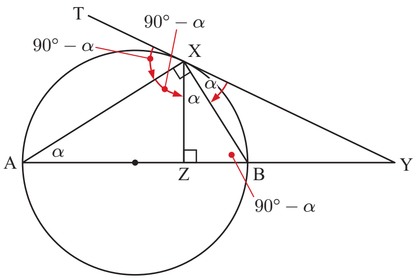
$= -1 \times 2$

$= -2$

$\therefore\ \mathrm{T_B} \circ \mathrm{T_A}$ reverses sense and doubles area.

page 293 **Worked Solutions EXERCISE 2D Question 20**, should correctly calculate $X\widehat{A}Z$:

20



We join [XA] and [XB].

Let $Z\widehat{X}B = \alpha$

$A\widehat{X}B = 90°$ {angle in a semi-circle}

$\therefore$ $Z\widehat{X}A = 90° - \alpha$

$\therefore$ $X\widehat{A}Z = 180° - 90° - (90° - \alpha) = \alpha$ {angles in a triangle}

$\therefore$ $B\widehat{X}Y = \alpha$ {angle between tangent and chord}

$Z\widehat{B}X = 180° - 90° - \alpha = 90° - \alpha$ {angles in a triangle}

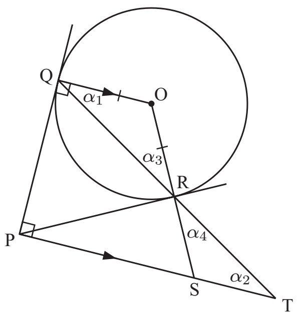
$\therefore$ $A\widehat{X}T = 90° - \alpha$ {angle between tangent and chord}

$\therefore$ $Z\widehat{X}B = B\widehat{X}Y = \alpha$ and $Z\widehat{X}A = A\widehat{X}T = 90° - \alpha$

$\therefore$ [XB] bisects $Z\widehat{X}Y$ and [XA] bisects $Z\widehat{X}T$.

page 293 **Worked Solutions EXERCISE 2D Question 24**, should read:

24



We join [OQ]. $O\widehat{Q}P = 90°$ {radius-tangent}

pages 308 and 309 **Worked Solutions EXERCISE 2K.1 Question 3**, should be replaced entirely with:

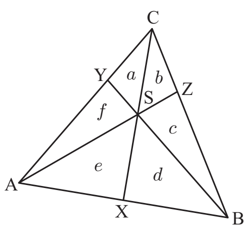
3 **a** By Ceva's theorem, $\dfrac{AX}{XB} \times \dfrac{BZ}{ZC} \times \dfrac{CY}{YA} = 1$

$\therefore$ $\dfrac{AX}{XB} \times 2 \times \dfrac{2}{3} = 1$

$\therefore$ $\dfrac{AX}{XB} = \dfrac{3}{4}$

$\therefore$ X divides [AB] in the ratio 3 : 4.

b



Now $c = 2b$

$f = \frac{3}{2}a$

$d = \frac{4}{3}e$

{areas of triangles are proportional to their bases if altitudes are equal}

Similarly, $e + d + c = 2(f + a + b)$

$\therefore$ $e + \frac{4}{3}e + \cancel{2b} = 3a + 2a + \cancel{2b}$

$\therefore$ $a = \frac{7}{15}e$ and so $f = \frac{7}{10}e$

Also, $4(a + f + e) = 3(b + c + d)$

$\therefore$ $4(\frac{7}{15}e + \frac{7}{10}e + e) = 3(b + 2b + \frac{4}{3}e)$

$\therefore$ $\frac{26}{3}e = 4e + 9b$

$\therefore$ $b = \frac{17}{21}e$

$\therefore$ AS : SZ $= f + a : b$

$= \frac{7}{15}\cancel{e} + \frac{7}{10}\cancel{e} : \frac{17}{21}\cancel{e}$

$= 49 : 34$